erschienen in: Danson M. (ed), London 1997, Regional Governance and Economic Development
European Research in Regional Science 7, pp. 210-228 (ISBN 0-85086-167-5)

Regional growth and convergence in a tax-sharing system

K Kellermann
University of Cologne
H Schmidt
University of Zurich

1 Introduction

In most existing federations tax revenue is redistributed among the federal levels, the states, and the local jurisdictions. In Germany this *tax-sharing system* (TSS) has a strong levelling effect. The 'Länder' share their tax revenue more or less equally in order to pursue "unitary living standards in the federal territory", as required by the German constitution. Germany's reunification has brought the inclusion of the relatively poor East German Länder into the existing TSS of West Germany. This has increased the magnitude of the interregional transfers and has therefore attracted new interest in this aspect of German public finance. For example, in 1995 East German Saxonia had only a quarter of the tax revenue per inhabitant as did Hamburg, Germany's richest federal state, before taxes were redistributed.(1) After redistribution, the Saxonian fiscal endowment per inhabitant reached nearly 100% of Hamburg's.(2) Intergovernmental transfers in the European Union do not have any comparable levelling effect. However, such redistributional measures and their effects might gain importance in the light of the planned Economic and Monetary Union.

In this paper we contribute to the debate on the sense and meaning of revenue sharing. We analyse the effects of a TSS, approximating the German example, on the growth and convergence process of regions, that is, our analysis is carried out in a dynamic setting. The static allocative effects of tax sharing or intergovernmental grants are described by Oates (1972), Stiglitz (1977), and Wildasin (1986). In the case of locally provided public goods, external benefits to other jurisdictions can lead to an underprovision of public services. On normative grounds appropriate matching grants will induce state or local governments to internalise the benefits, provided to residents of other jurisdictions, into the local decision calculus. Stiglitz's analysis justifies intergovernmental transfers if they can prevent 'inefficient migration'. The argument hinges on the assumption of an immobile local production factor. Our paper shows that the discussion might be enriched by taking into account both short-run and long-run effects on interregional distribution and allocation. By focusing the analysis on economies in transition we might deduce statements on the effects of compensating payments between 'old' and 'new' German Länder.

The redistribution of tax revenues among jurisdictions influences the possibilities of regional governments to supply productive goods and services. Thereby the TSS has effects on the development of productivities in the private sectors of the regions. Thus, it is conceivable that regional governments whose budgets will rise by receiving tax transfers are able to supply more productive inputs, thereby increasing productivity of private factors and output. Therefore, although the focus is on the redistribution of tax revenue among regional governments, we can analyse repercussions of this mechanism on the interpersonal distribution of income among citizens of different regions.(3)

Our analysis is of a normative macroeconomic nature. Important politico-economic aspects of a TSS, as analysed, for example, by Oates (1979), Gramlich (1987), and Inman (1988), are ignored. The theoretical basis is a one-sector neoclassical growth model with public services or goods as productive inputs. In other words, public goods and services are intermediate goods (compare Kaizuka, 1965). These inputs enter the production function as a *flow* (compare Barro, 1990). This aspect distinguishes our model from the contributions of Arrow and Kurz (1970), Aschauer (1989; 1995), and others, where the *stock* of public capital is considered as a primary factor of production. The assumption that the public sector enters the aggregate production function of the private sector with a *flow* of goods and services is crucial for our results.

We will compute the growth rates and levels of local productivities, both without and with interregional financial transfers, as well as the output gains and losses of such a redistributive policy. Thus we can identify and compare both the allocative and the distributive effects of interregional redistribution. If government spending is productive, the giving regions will certainly lose in terms of foregone output. However, will they lose as much as the receiving regions gain? Put differently, what is the effect on the level and growth of total output of the federal economy?

To see the peculiarities of our model it is helpful to compare it with the more common models where public capital contributes to private productivity. In those models private capital is assumed to be fully mobile, and the main reason for an interregionally inefficient allocation of resources is the immobility of the public capital stock. Under these assumptions, the redistribution of public investment from a capital-rich region to a capital-poor region may lead to a better allocation of aggregate capital and therefore to instantaneous gains for the federal economy (Homburg, 1993). This viewpoint can be criticized in two ways. First, we do not think of public capital as the one and only type of publicly provided input in private production.

(1) According to the 'Finanzausgleichsgesetz' (law on tax sharing) and before distribution of the value-added tax.

(2) Neglecting grants from the 'Bund', the federal government, according to Articles 104a and 91a of the 'Grundgesetz', the German constitution.

(3) Bayoumi and Masson (1995) empirically estimate the effects of fiscal flows within the United States and Canada on personal incomes. They find that flows of taxes, transfers, and grants reduce long-term income inequalities by 22 cents in the dollar in the USA and by 39 cents in dollar in Canada.

Moreover, the government uses labour services and capital goods or services to carry out public production. Second, we think it is wrong to trace back interregional differences in private capital endowments primarily to differences in public capital stocks. Various empirical analyses of regional convergence show that capital mobility is low even among regions within a country (Barro and Sala-i-Martin, 1995; Seitz, 1995). Our analysis allows for these aspects.

The main outcome of our theoretical analysis is that the provision of more productive government services in a (capital-)poor region and fewer of these services in a (capital-)rich region leads to a (temporary) loss in the *level* of aggregate output *if* capital mobility is not infinite. Thereby, the result which could be expected in a politico-economic framework would be supported by our pure macroeconomic analysis. However, as will be shown, these losses will be quite small compared with the distributional effects of the shift in resources. Therefore, a first conclusion will be that tax sharing can be an effective means of redistribution. Moreover, the most interesting result appears when considering the dynamic allocative effects of a TSS in our framework. In the medium term the redistribution may even lead to positive growth effects in the economy. Therefore, the respective time horizon is important for drawing conclusions in our model.

In section 2 we lay out our basic neoclassical growth model for an isolated region. The transition to the steady state and its properties are analysed. In particular, we consider the process and the speed of convergence by which the region approaches its long-run equilibrium. Moreover, an optimal tax rate is calculated and interpreted. In section 3, we consider two such regions, which form a federal state and are connected by a TSS. At the time of implementation of the TSS the two regions will be on different stages of development. The fiscal transfers do take place on the governmental level such that the budget constraints of the local governments are altered when the TSS is implemented. The effects of the system change on investment, growth, the speed of convergence, the optimal tax rate, and the steady state are analysed. Numerical simulations are carried out to illustrate the effects. Section 4 concludes.

2 The basic model

We start our analysis by considering a single, isolated region. It is characterised by a per capita production function of the form

$$y_t = k_t^{\alpha} g_t^{\beta}, \qquad \text{with} \quad \alpha + \beta < 1. \tag{1}$$

Here y_t is output per capita, k_t is capital intensity, and g_t are productive public expenditures per capita at time t, that is, we do not consider the stock of public infrastructure as an extra input in the production function but rather the flow of publicly provided services. Note that they are assumed to have a 'private good' character (compare Bergstrom and Goodman, 1973; Borcherding and Deacon, 1972;). This assumption leads to the inclusion of public services in per capita form in equation (1). We adapted this speci[illegible]tion from Barro (1990), who assumed $\alpha + \beta = 1$ in order to generate long-run (endogenous) growth. However, with this assumption no transition dynamics, that is, no convergence, occurs in the model (compare subsection 3.2.2). Throughout the paper we neglect technical progress and population growth. The latter implies that the labour force is also assumed constant.

The government is faced with the budget constraint

$$g_t = \tau y_t, \qquad \text{with} \quad \tau \leqslant 1, \tag{2}$$

that is, τ is a constant tax rate on output and we constrain the government to run a balanced budget every period. The production function can therefore be expressed as

$$y_t = k_t^{\alpha/(1-\beta)} \tau_t^{\beta/(1-\beta)}. \tag{3}$$

The transitional dynamics are characterised by the accumulation of capital. For reasons of simplicity we refrain from an analysis of the optimal growth path (compare Ramsey, 1928) but assume that private agents save and invest a certain share, s_t, of disposable income in every period t. In accordance with Solow's (1956) original contribution and with what the data show we can even think of an intertemporal constant s. Nevertheless, many implications of the neoclassical growth model are the same in the Solow and the Ramsey specifications.

Capital accumulation takes place as investment less the depreciation, δk, of capital:

$$\dot{k}_t = s(1-\tau)y_t - \delta k_t, \qquad \text{where} \quad \dot{k}_t \equiv \frac{dk_t}{dt}. \tag{4}$$

Thus, the growth rate of per capita capital stock in the transitional dynamics is given by:

$$\hat{k}_t = s(1-\tau)k_t^{(\alpha+\beta-1)/(1-\beta)} \tau^{\beta/(1-\beta)} - \delta, \tag{5}$$

where a $\hat{}$ denotes a growth rate, that is, for instance $\hat{k} = \dot{k}/k$. The economy's growth rate is proportional to this expression: by differentiating equation (3) we get $\hat{y} = [\alpha/(1-\beta)]\hat{k}$. In the long run the capital intensity converges towards a constant value k^*. By letting the growth rate in equation (5) be equal to zero, we get[4]

$$k^* = \left[\frac{s(1-\tau)\tau^{\beta/(1-\beta)}}{\delta}\right]^{(1-\beta)/(1-\alpha-\beta)}. \tag{6}$$

[4] If we considered exogenous (labour-augmenting) technical progress ($\hat{A}$) and a constant population growth rate ($\hat{L}$) in the model we could express productivity and capital intensity in units of effective labour, that is, $k^e = K/(AL)$ and $y^e = Y/(AL)$. Then we could include both rates in the formulas for the (transitional) growth rate to account for 'capital widening', that is, the parameter δ would not only include depreciation but also $\hat{A}$ and $\hat{L}$ [compare, for example, Barro and Sala-i-Martin (1995, chapter 2)]. No essential r[illegible] or conclusion would change.

2.1 Optimal taxation

The effect of the tax rate, τ, on the growth rate and the steady state value k^* is ambiguous. Through the term $(1 - \tau)$, which determines the share of disposable income for private individuals, τ has a negative effect. Through the term $\tau^{\beta/(1-\beta)}$, the tax rate influences transitional growth and steady-state level positively. Therefore, there exists some 'optimal tax rate' τ^* which maximises growth, net investment, and consumption in the transitional dynamics, as well as labour productivity and steady-state consumption.[5] Therefore, the crucial question of intertemporal resource allocation addressed by Ramsey (1928) becomes rather simple. A benevolent government will choose a tax rate that maximises current consumption to yield current utility. The same tax rate will simultaneously maximise investment and, therefore, future production and consumption (see Chiang, 1992, page 111). Thus, by choosing an appropriate tax rate τ^* the government not only maximises present consumption but also consumption in later periods. Maximisation of $\hat{k}$ or k^* with respect to τ gives:

$$\tau^* = \beta. \tag{7}$$

This result is a kind of "*golden (tax) rule*" for our economy. In the optimum, the share of tax revenue in domestic product has just to be equal to the output elasticity of government expenditure. Note that this golden rule is not only valid in the steady state but in transitional dynamics, too. Barro (1990) shows that it holds also in a model with endogenous growth, that is, in a setting where $\alpha + \beta = 1$, with positive steady-state growth and no transitional dynamics.

2.2 Speed of convergence

The rate of convergence towards the steady state is given as the percentage by which the gap remaining to the long-run equilibrium is closed during the present period. In the related literature (compare, for example, Mankiw et al, 1992) this rate is usually calculated by (Taylor-)approximating the fundamental differential equation(s) of the model around the steady state. Here, this is the accumulation equation for the capital stock k as given by equation (4). The phase diagram for the $\dot{k}$ function is shown in figure 1.

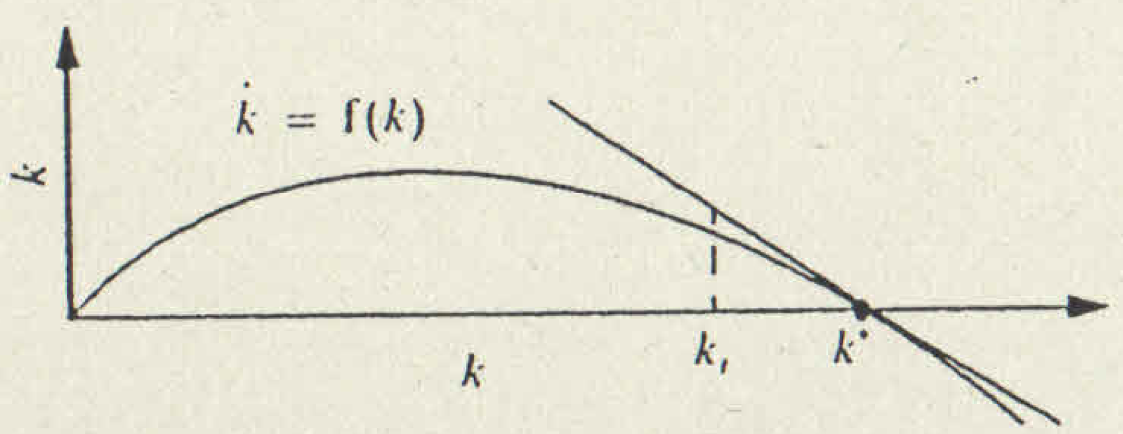

Figure 1. The speed of convergence.

In this case, the speed of convergence λ is given by (the negative of) the slope of $\dot{k}$ in k^*:

$$\lambda = \delta \frac{1 - \alpha - \beta}{1 - \beta}, \tag{8}$$

that is, the gap between the current capital intensity and k^* closes at the rate of λ, approximately, if the economy is close to the steady state. The approximated speed of convergence is completely determined by the exogenous parameters of the model. In particular, it is independent of the tax rate τ and the savings rate s. Note, however, that this convenient property of the model does not hold when we consider stages of development which are not in the neighbourhood of the steady state (where the Taylor approximation is applicable), as we do in the following section.

3 Financial transfers in a two-regions model

3.1 Modification of the one-region model

In this section the analysis of the last section is modified and extended. We assume that there are two regions, of the kind described in section 2, which form a federal state with a federal government. We want to determine the effects of a TSS on the growth and convergence process in both regions. Therefore it is useful to assume that the two regions are identical (even in population size) with only one exception: region 1 has a lower capital intensity than region 2 at the time of introduction of the TSS.

Throughout the paper we do not allow for factor mobility among the regions. This rather strong assumption not only allows for simplification but makes our case a benchmark in the sense that the only way to accelerate the convergence and catching-up process of poorer regions is by the redistribution of tax revenue. To justify this assumption we consider the extreme cases. On the one hand, with fully mobile factors our neoclassical growth model would predict an instantaneous catching up of the poorer region to the richer one and the TSS would lose any meaning. Obviously, this prediction drastically contradicts reality even for regions which are well connected, such as the two parts of Germany. On the other hand, to rule out factor mobility (over)emphasises the effects of the TSS on the growth path of the economies. However, these effects will not vanish as long as there are *any* restrictions to factor mobility. Therefore, we choose the latter case and keep in mind that the effects may be blurred by mobile factors.[6]

Transfers among regions are designed in a way as to equalise regional fiscal power, that is, each of the two regions gets one half of total tax revenue (because population size is the same). Moreover, transfers have the character of conditional grants. Transfers as well as the regions' own tax revenue have to be used by the government for productive services only. Formally this changes the budget restriction in the following way:

$$g_t^i = \tfrac{1}{2}(\tau y_t^1 + \tau y_t^2), \qquad \text{for} \quad i = 1, 2. \tag{9}$$

[5] Consumption in period t is $c_t = (1 - \tau)(1 - s)y_t = (1 - \tau)(1 - s)\,\tau^{\beta/(1-\beta)} k_t^{\alpha/(1-\beta)}$.

[6] Barro et al (1995) construct a neoclassical growth model where capital is partially mobile. They show that this will correspondingly increase the speed of convergence. However, as in our case, the economies will not converge infinitely fast as long as there is some immobili[illegible]

By application of this rule, region i's production level in period t now depends on the level of the other region's taxable income. With tax-revenue-equalising transfers we find that $g_t^1 = g_t^2$. By that, equation (9) transforms to

$$g_t^i = \tfrac{1}{2}\tau\left[(k_t^1)^{\alpha}(g_t^i)^{\beta} + (k_t^2)^{\alpha}(g_t^i)^{\beta}\right],$$

and therefore,

$$g_t^i = \{\tfrac{1}{2}\tau[(k_t^1)^{\alpha} + (k_t^2)^{\alpha}]\}^{1/(1-\beta)}. \tag{10}$$

Substitution into equation (1) yields the production function for a region which is involved in a TSS with another region, according to the rule in equation (9).

For the moment consider one region explicitly: for example, region 1's production function can then be expressed as:

$$y_t^1 = (k_t^1)^{\alpha/(1-\beta)}(\tfrac{1}{2}\tau)^{\beta/(1-\beta)}\left[1 + \left(\frac{k_t^2}{k_t^1}\right)^{\alpha}\right]^{\beta/(1-\beta)}. \tag{11}$$

Compared with the production function of an isolated region [see equation (3)], per capita output of a region involved in a TSS depends additionally on the stage of development (k_t^2) of the other region. More specifically, a larger k_t^2/k_t^1 ratio will give a higher output level for region 1.

At this stage it is useful to take a look at the production function when the regions differ in size, that is, in the size of their labour force L_i. Then region 1's production function is

$$y_t^1 = (k_t^1)^{\alpha/(1-\beta)}\tau^{\beta/(1-\beta)}\left\{\left[1 + \frac{L_2}{L_1}\left(\frac{k_t^2}{k_t^1}\right)^{\alpha}\right]\Big/\left(1 + \frac{L_2}{L_1}\right)\right\}^{\beta/(1-\beta)}. \tag{12}$$

The derivative of y^1 with respect to L_2/L_1 shows that per capita production of the poorer region 1 is the larger the smaller the region is compared with region 2. Ceteris paribus, the average payoff for every individual in the poor region increases as the number of people who produce and pay taxes in the rich region increases. However, this will qualify our results only quantitatively (see subsection 3.2.2), so we suppress this aspect again and return to the simplifying assumption of equal sizes.

The growth rate of region 1's capital intensity in the presence of a TSS is given by

$$\hat{k}_t^1 = s(1-\tau)(k_t^1)^{(\alpha+\beta-1)/(1-\beta)}(\tfrac{1}{2}\tau)^{\beta/(1-\beta)}\left[1 + \left(\frac{k_t^2}{k_t^1}\right)^{\alpha}\right]^{\beta/(1-\beta)} - \delta. \tag{13}$$

As with the production level, the growth rate is larger the higher the capital intensity ratio k_t^2/k_t^1. Change of the region's indices in equations (11), (12), and (13) will lead to the corresponding expressions and respective conclusions for region 2.

Comparison of production levels and growth rates without TSS versus with TSS, respectively, [equations (3) versus (11), and (5) versus (13)], shows that

$$y_{t,\mathrm{TSS}}^1 > y_{t,\neg\mathrm{TSS}}^1, \quad \text{and} \quad \hat{k}_{t,\mathrm{TSS}}^1 > \hat{k}_{t,\neg\mathrm{TSS}}^1, \quad \text{if} \quad \frac{k_t^2}{k_t^1} > 1 \quad \text{and vice versa,}$$

where a '$\neg$' means 'without'. Thus, income and growth rate of a region with a TSS will increase compared with a situation without TSS if the region's capital intensity is smaller than in the other region. This result is rather obvious, because the capital-poor region will receive additional tax revenue with a TSS, which raises its production possibilities.(7) Accordingly, the growth rate of the capital-rich region will decrease as long as it remains the capital-rich region, that is, until the capital-poor region has caught up.

3.1.1 *Speed of convergence*

How does the introduction of a TSS change the speed of convergence to the steady state? Unlike in the analysis for an isolated region we cannot compare the two regions by linearly approximating their respective $\dot{k}$ functions around the steady state, for two reasons: first, we would like to consider phases of development which are rather far away from the steady state, where a linear approximation may not be suitable any more; second, and more important, with the same production functions, savings behaviour, population growth, etc. the steady state will be the same for both regions and independent of the existence of a TSS, that is, it will be the *same as for an isolated region* such as the one we have analysed in section 2. This is because the region with the initially lower capital intensity will have caught up with the other region in the steady state, which will make the TSS meaningless. Thus, k^* of expression (6) is still valid with a TSS and is valid for both regions, which leaves the approximated convergence speed untouched by the implementation of a TSS. Therefore, we have to use another, more pragmatic approach for calculating the convergence speed, λ, of a region in a TSS.

$$\lambda_t^i = \frac{\dot{k}_t^i}{k^* - k_t^i} = \frac{k_t^i\hat{k}_t^i}{k^* - k_t^i}. \tag{14}$$

That is, λ_t^i is the change in capital intensity relative to the gap between the instantaneous capital intensity and the long-run equilibrium. Geometrically (see figure 1) λ_t^i is the tangent of the angle between the k axis and an imaginary line between the instantaneous k value and k^*. The rate of closing the gap to the steady state increases over time, as the k curve is concave. Thus, we tend to underestimate the average speed of convergence. However, the constant λ we obtained in section 2 [see expression (8)] is also imperfect because it tends to overestimate the average convergence speed, as we linearised the $\dot{k}$ curve around k^* to get λ. Therefore, the two measures cannot be compared directly but they do give the range of the convergence rate as the economy moves towards the steady state.

As a TSS increases and decreases $\dot{k}_t^i$ in both regions, respectively, the convergence to the long-run equilibrium speeds up or slows down depending on whether the region is poorer or richer in terms of per capita income at the time

(7) What is less obvious is the size and the net outcome of the effects of a TSS which we will consider below (compare section 3.2).

of introduction of the TSS (for a graphical illustration see figures 2 and 4). For the time-varying convergence speeds we have the following relationships between the convergence speed without a TSS and the speeds of the receiving region 1 and the giving region 2 in the presence of a TSS at a given k:

$$\lambda^1_{\mathrm{TSS}} \geqslant \lambda_{\neg\mathrm{TSS}} \geqslant \lambda^2_{\mathrm{TSS}}\,,$$

where the equality signs become valid only in the steady state.

3.1.2 *Closing the income gap*

As the growth rates of the poor and the rich regions are increased and decreased by a TSS, respectively, we may notice that the catch-up process of labour productivity will be speeded up with a TSS. As we pointed out in the introduction, this is the main if not the sole reason for having a TSS in various countries. Equivalent to measuring the speed of convergence to the steady state, the catching up can be measured by the (percentage) rate by which the labour productivity gap between the two regions vanishes. Formally, and in discrete terms, we calculate the rate, μ, by which the poor region 1 is catching up with rich region 2 as

$$\mu = 1 - \frac{y_t^2 - y_t^1}{y_{t-1}^2 - y_{t-1}^1}\,. \tag{15}$$

By substituting the period of introduction of the TSS (period 0) for $(t-1)$, formula (15) may be used to calculate the part of the initial gap between the two regions that is closed after t periods of the TSS. However, the catch-up process will be completed only in the joint steady state of the regions. Note again that in our model the regions will stop growing in the steady state (or grow with some exogenously given rate of technical progress, a parameter which we have neglected in our specification).

3.2 Simulation of the transitional dynamics

The time paths of the capital intensity levels in period t are given by

$$k_t^i = k_0^i \exp\int_0^t \hat{k}_T^i \,\mathrm{d}T, \quad i = 1,2, \tag{16}$$

where k_0^i is capital intensity at some initial date and $\hat{k}_T^i$ is the growth rate of the capital intensity at time T. Note that in the transitional dynamics the growth rate changes continuously. It is determined by

$$\hat{k}_T^1 = s(1-\tau)\left(\frac{y_T^1}{k_T^1}\right) - \delta, \tag{17}$$

as in equations (5) and (13), respectively.

Now we are able to express the time path of production in terms of initial conditions and given parameters. For example, region 1's per capita income in period t, if it is isolated, is

$$y_t^1 = \left(k_0^1 \exp\int_0^t \hat{k}_T^1 \,\mathrm{d}T\right)^{\alpha/(1-\beta)} \left(\tfrac{1}{2}\tau\right)^{\beta/(1-\beta)}, \tag{18}$$

and if it is tied to region 2 by a TSS it is

$$y_t^1 = \left(k_0^1 \exp\int_0^t \hat{k}_T^1 \,\mathrm{d}T\right)^{\alpha/(1-\beta)} \left(\tfrac{1}{2}\tau\right)^{\beta/(1-\beta)} \times\left\{1 + \left[k_0^2 \exp\int_0^t \hat{k}_T^2 \,\mathrm{d}T \Big/ \left(k_0^1 \exp\int_0^t \hat{k}_T^1 \,\mathrm{d}T\right)\right]^{\alpha}\right\}^{\beta/(1-\beta)}. \tag{19}$$

Again, a change of the region's indices will give the corresponding expressions for region 2.

We would like to get the exact time paths of capital intensity and productivity by explicitly solving these equations for k_t^i and y_t^i, respectively. But because of the growth rate in the exponentials on the right-hand side of the equations, this is impossible. However, for the isolated region (and because of the simple Cobb–Douglas form of the production function) we could bypass this problem. For example, reducing equation (5) to a linear differential equation and solving it will deliver a closed-form solution for k_t^i that only depends on some initial value k_0^i and the parameters of the model [see Barro and Sala-i-Martin (1995, page 53) for the derivation in a similar model]. However, for the two-region economy with a TSS of section 3.1 such a quantitative solution of the time paths is impossible. With equation (13) and its counterpart for region 2 we have a simultaneous system of nonlinear differential equations which is not solvable. Therefore, we will demonstrate the development of per capita income (and the capital intensity) over time in a numerical simulation.

3.2.1 *Basis simulation*

We choose some usual values for the parameters of the model. For the production elasticity of capital α we take $\frac{1}{3}$, as this is approximately the share of capital in total income. An average ratio of tax revenue to gross domestic product, for example, in Germany, is approximately 0.2. Therefore, the tax rate, τ, and the production elasticity, β, of government services, are assumed to be 0.2. The underlying assumption here is that the government has chosen the optimal tax rate. For s we take the average rate of private investment, which is approximately 0.2. Last, for the depreciation rate we assume the value 0.06. Note that by choosing a typical yearly depreciation rate of 6% in our simulations the length of one period should be considered as one year. To sum up, the parameter values in our basic simulation are:

$$\tau = \beta = 0.2, \quad \alpha = \tfrac{1}{3}, \quad s = 0.2, \quad \delta = 0.06\,.$$

These values determine the steady-state position of our regions, for example, k^* will be reached at a value of 2.7 [compare with equation (6)].

To see the development of y over time in a setting with a TSS we use the following discrete time expressions for our simulation [compare equation (20) with equation (11)],

$$y_t^i = (k_t^i)^{\alpha}\left(\tfrac{1}{2}\tau\right)^{\beta/(1-\beta)}[(k_t^1)^{\alpha} + (k_t^2)^{\alpha}]^{\beta/(1-\beta)}\,, \tag{20}$$

where

$$k_t^i = k_{t-1}^i + \dot{k}_{t-1}^i\,, \tag{21}$$

with $\dot{k}_{t-1}^i$ $(= \hat{k}_{t-1}^i k_{t-1}^i)$ according to equation (13). As a reference we simulate the transitional dynamics for the case without a TSS, that is, we use equation (3) to determine the output level in every period t as well as the corresponding expression for capital intensity [by means of equation (4)].

Last, we have to assume initial values for the region's capital intensities. We choose:

$$k_0^1 = 1, \qquad \text{and} \qquad k_0^2 = \frac{k_0^2}{k_0^1} = 2.$$

The corresponding values of y_0^1 and y_0^2 are calculated according to equation (3) in our basic model for the isolated region. We assume that if a TSS as implemented it would be effective from period 1 onwards.

Figure 2 exhibits per capita income y and capital intensity k on its axes. Production y and the share of production saved and invested by the private agents, $s(1-\tau)y$, for both regions considered in isolation are plotted according to the assumed parameters and functional forms. The depreciation is illustrated by the δk line. The regions start out at some k, grow by the difference $[s(1-\tau)(y/k)-\delta]$ [compare equations (3) to (5)], and converge to the steady state where this difference is zero. With the parameters given, the steady state is reached when $k^* = 2.7$, $s(1-\tau)y^* = 0.16$, and $y^* = 1.02$. As regions 1 and 2 are at $k_0^1 = 1$ and $k_0^2 = 2$, respectively, when a TSS is implemented, the production functions in figure 2 will shift upwards for the poor region 1 (y_{TSS}^1) and downwards for the rich region 2 (y_{TSS}^2). The savings function, $s(1-\tau)y$, will shift accordingly (for reasons of clarity this is not shown in the figure). As we have also seen formally, this means a higher per capita income for every k for region 1, and a lower one for region 2 for the rest of their way to the steady state.

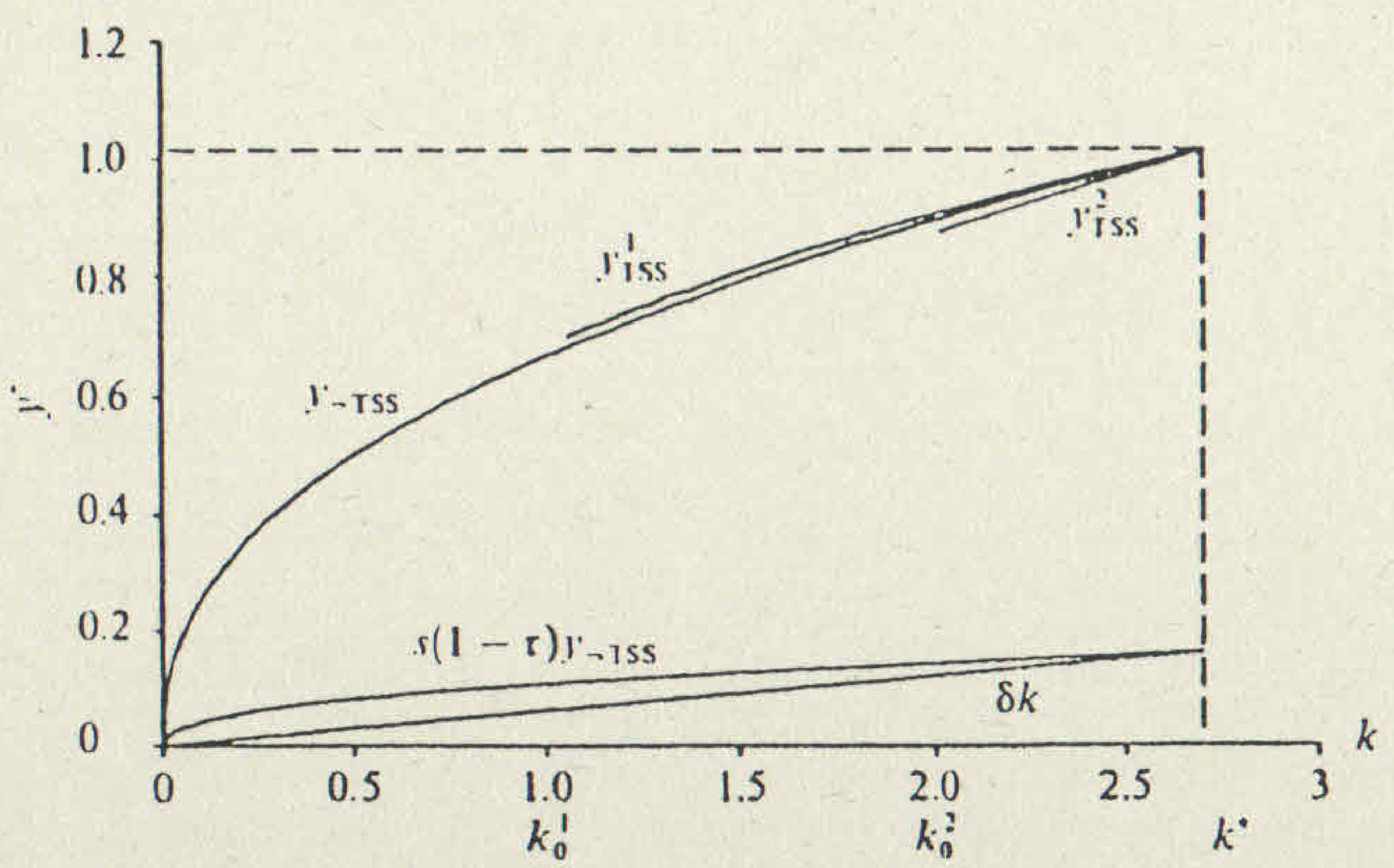

Figure 2. Shifts in the per capita production function y because of the TSS.

A crucial question for the federal government is whether the output gains for region 1 can outweigh the losses for region 2. The simulation with the values from above is shown in table 1 (over) for the first 65 periods after introduction of the TSS. The values for the respective development without the TSS are also listed. In the 10th and 11th columns the gains and losses for the two regions are given in terms of the cumulative income they would have achieved in periods 0 to t had the TSS not been implemented. Region 1's gains are considerable and amount to more than 2% for a long and relevant period of time, whereas the percentages of the rich region 2 follow a similar path but to the negative and with slightly lower absolute values.

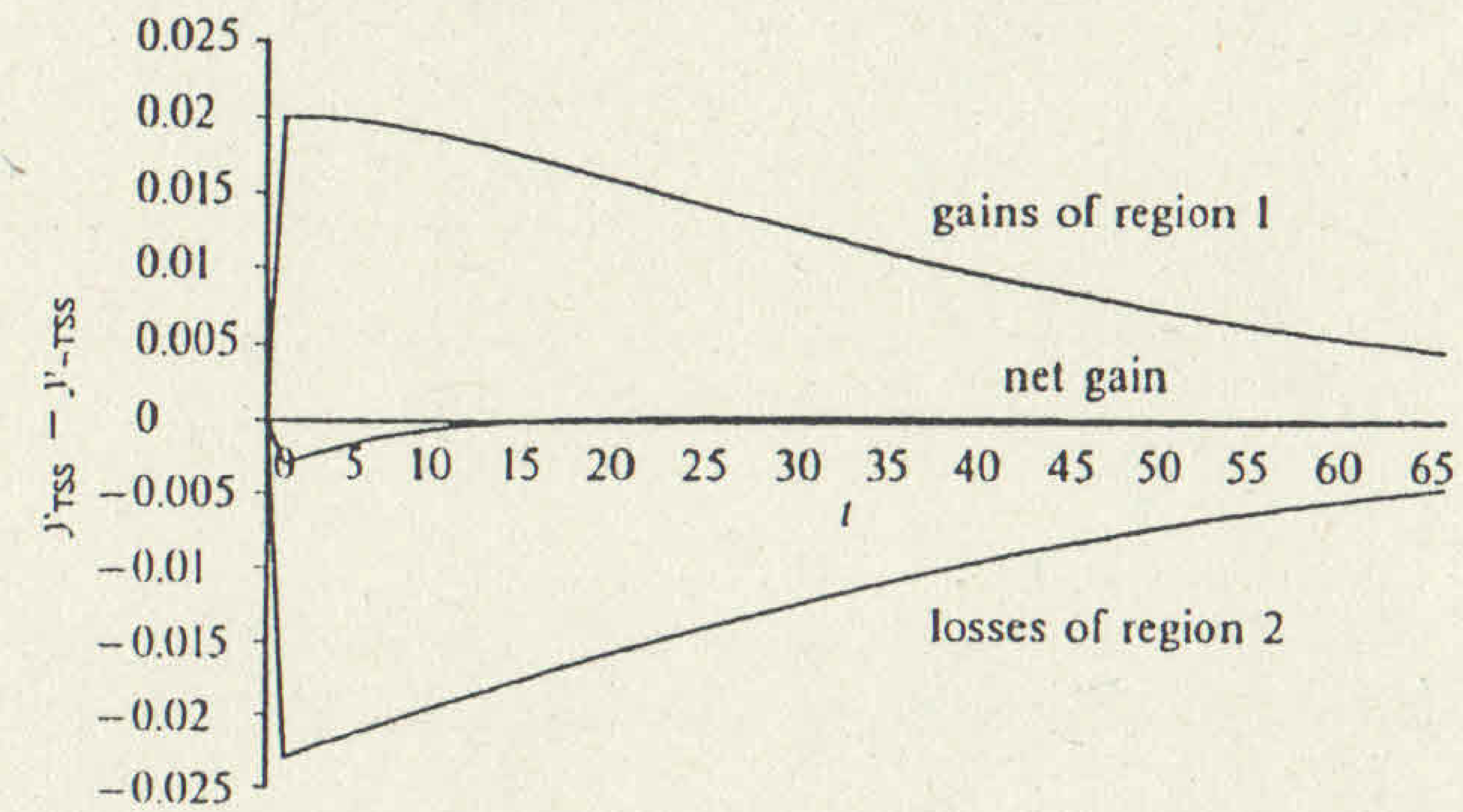

Figure 3. Gains and losses in per capita production y through the TSS.

The noncumulated output gains for both regions as well as the net gain for the federal state are plotted in figure 3. Compared with the situation without the TSS the federal state is worse off in the first periods after the introduction of the TSS. This is because the redistribution of tax revenue is inefficient, ceteris paribus. Part of region 2's taxes are transferred to region 1, where they are less productive, as $k_1 < k_2$, but the level of g is the same everywhere:[8]

$$\frac{\partial y_1}{\partial g} = k_1^\alpha \beta g^{\beta-1} < k_2^\alpha \beta g^{\beta-1} = \frac{\partial y_2}{\partial g}\,, \tag{22}$$

that is, the transfers induce a *negative level effect* on aggregate disposable income. However, this negative level effect is counteracted by a *positive growth effect*. The tax redistribution induces higher private investment in region 1 and less investment in region 2 because of the resulting changes in disposable incomes. Seen from an aggregate point of view, this means a more efficient allocation of investment, as marginal product of capital is higher in region 1 than in region 2:

$$\frac{\partial y_1}{\partial k_1} = \alpha k_1^{\alpha-1} g^\beta > \alpha k_2^{\alpha-1} g^\beta = \frac{\partial y_2}{\partial k_2}\,. \tag{23}$$

[8] Note that without TSS the marginal productivities of g are constant and equal across regions: $\partial y_1/\partial g_1 = \;\;/\tau = \partial y_2/\partial g_2$.

t	With TSS				Without TSS				region 1 in %[b]	region 2 in %[b]	for federal state in %[c]	with TSS	without TSS
	k^1	y^1	k^2	y^2	k^1	y^1	k^2	y^2					
0	1	0.6687	2	0.8927	1	0.6687	2	0.8927	0	0	0	0	0
1	1.0470	0.7017	2.0228	0.8739	1.0470	0.6817	2.0228	0.8969	2.94	−2.56	−0.1866	23.1	3.9
2	1.0964	0.7141	2.0413	0.8785	1.0932	0.6940	2.0450	0.9010	2.91	−2.53	−0.1711	26.6	7.6
3	1.1449	0.7259	2.0594	0.8828	1.1387	0.7059	2.0664	0.9049	2.88	−2.50	−0.1569	29.9	11.1
4	1.1924	0.7372	2.0771	0.8870	1.1833	0.7173	2.0872	0.9087	2.86	−2.47	−0.1441	33.1	14.5
5	1.2388	0.7480	2.0944	0.8911	1.2271	0.7283	2.1074	0.9123	2.83	−2.44	−0.1324	36.1	17.8
6	1.2841	0.7584	2.1113	0.8951	1.2700	0.7388	2.1269	0.9158	2.80	−2.41	−0.1218	38.9	20.9
7	1.3284	0.7683	2.1278	0.8989	1.3120	0.7489	2.1458	0.9192	2.77	−2.38	−0.1121	41.7	23.9
8	1.3716	0.7777	2.1440	0.9026	1.3531	0.7585	2.1641	0.9225	2.74	−2.35	−0.1032	44.2	26.8
9	1.4138	0.7868	2.1597	0.9061	1.3933	0.7678	2.1819	0.9256	2.70	−2.33	−0.0951	46.7	29.5
10	1.4548	0.7955	2.1751	0.9096	1.4325	0.7768	2.1991	0.9287	2.67	−2.30	−0.0877	49.0	32.2
11	1.4948	0.8038	2.1902	0.9129	1.4709	0.7854	2.2157	0.9316	2.64	−2.27	−0.0810	51.3	34.7
12	1.5337	0.8117	2.2048	0.9161	1.5083	0.7936	2.2318	0.9344	2.61	−2.24	−0.0747	53.4	37.1
13	1.5716	0.8194	2.2191	0.9193	1.5448	0.8016	2.2474	0.9371	2.58	−2.22	−0.0690	55.4	39.5
14	1.6084	0.8267	2.2330	0.9223	1.5803	0.8092	2.2625	0.9397	2.54	−2.19	−0.0638	57.3	41.7
15	1.6442	0.8337	2.2466	0.9252	1.6150	0.8166	2.2771	0.9423	2.51	−2.17	−0.0589	59.2	43.9
16	1.6789	0.8405	2.2599	0.9280	1.6487	0.8236	2.2913	0.9447	2.48	−2.14	−0.0545	60.9	45.9
17	1.7127	0.8470	2.2727	0.9307	1.6816	0.8304	2.3049	0.9470	2.45	−2.11	−0.0504	62.6	47.9
18	1.7454	0.8532	2.2853	0.9334	1.7136	0.8370	2.3182	0.9493	2.42	−2.09	−0.0466	64.2	49.8
19	1.7772	0.8591	2.2975	0.9359	1.7447	0.8433	2.3310	0.9515	2.39	−2.07	−0.0431	65.7	51.7
20	1.8080	0.8649	2.3094	0.9384	1.7749	0.8493	2.3433	0.9536	2.36	−2.04	−0.0399	67.2	53.4
21	1.8379	0.8704	2.3210	0.9408	1.8043	0.8552	2.3553	0.9556	2.33	−2.02	−0.0369	68.6	55.1
22	1.8669	0.8756	2.3322	0.9431	1.8329	0.8608	2.3669	0.9576	2.30	−1.99	−0.0342	69.9	56.8
23	1.8950	0.8807	2.3432	0.9453	1.8606	0.8662	2.3781	0.9594	2.27	−1.97	−0.0316	71.2	58.4
24	1.9222	0.8856	2.3539	0.9475	1.8876	0.8714	2.3889	0.9613	2.24	−1.95	−0.0293	72.4	59.9
25	1.9486	0.8903	2.3642	0.9496	1.9138	0.8764	2.3994	0.9630	2.21	−1.92	−0.0271	73.5	61.3
26	1.9741	0.8948	2.3743	0.9516	1.9392	0.8812	2.4095	0.9647	2.18	−1.90	−0.0251	74.6	62.7
27	1.9988	0.8991	2.3841	0.9535	1.9638	0.8859	2.4193	0.9663	2.15	−1.88	−0.0232	75.7	64.1
28	2.0228	0.9033	2.3936	0.9554	1.9877	0.8904	2.4287	0.9679	2.12	−1.86	−0.0214	76.7	65.4
29	2.0459	0.9073	2.4029	0.9572	2.0109	0.8947	2.4379	0.9694	2.10	−1.84	−0.0198	77.7	66.6
30	2.0683	0.9111	2.4118	0.9590	2.0334	0.8988	2.4467	0.9709	2.07	−1.82	−0.0183	78.6	67.8
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
50	2.3884	0.9637	2.5454	0.9844	2.3609	0.9565	2.5723	0.9913	1.61	−1.45	−0.0029	90.8	84.5
51	2.3993	0.9655	2.5502	0.9853	2.3723	0.9585	2.5766	0.9920	1.59	−1.43	−0.0026	91.1	85.0
52	2.4098	0.9671	2.5549	0.9861	2.3833	0.9603	2.5807	0.9927	1.57	−1.42	−0.0023	91.5	85.5
53	2.4199	0.9687	2.5594	0.9870	2.3939	0.9621	2.5847	0.9933	1.55	−1.40	−0.0020	91.8	86.1
54	2.4297	0.9702	2.5637	0.9878	2.4042	0.9638	2.5885	0.9939	1.54	−1.39	−0.0017	92.2	86.5
55	2.4392	0.9717	2.5679	0.9885	2.4142	0.9655	2.5923	0.9945	1.52	−1.37	−0.0014	92.5	87.0
56	2.4483	0.9732	2.5720	0.9893	2.4238	0.9671	2.5959	0.9951	1.50	−1.36	−0.0012	92.8	87.5
57	2.4571	0.9745	2.5760	0.9900	2.4331	0.9686	2.5993	0.9957	1.48	−1.35	−0.0010	93.1	87.9
58	2.4656	0.9758	2.5798	0.9907	2.4421	0.9701	2.6027	0.9962	1.46	−1.33	−0.0008	93.4	88.4
59	2.4738	0.9771	2.5835	0.9914	2.4508	0.9716	2.6059	0.9967	1.45	−1.32	−0.0006	93.6	88.8
60	2.4817	0.9783	2.5871	0.9920	2.4592	0.9729	2.6090	0.9972	1.43	−1.30	−0.0004	93.9	89.2
61	2.4894	0.9795	2.5906	0.9926	2.4673	0.9743	2.6120	0.9977	1.42	−1.29	−0.0002	94.1	89.5
62	2.4967	0.9807	2.5940	0.9932	2.4752	0.9756	2.6149	0.9982	1.40	−1.28	0.0000	94.4	89.9
63	2.5038	0.9818	2.5973	0.9938	2.4827	0.9768	2.6178	0.9986	1.38	−1.26	0.0001	94.6	90.3
64	2.5107	0.9828	2.6005	0.9944	2.4901	0.9780	2.6205	0.9990	1.37	−1.25	0.0003	94.8	90.6
65	2.5173	0.9838	2.6035	0.9950	2.4972	0.9792	2.6231	0.9995	1.35	−1.24	0.0004	95.0	90.9

Cumul., cumulative

[a] The following parameter values are used: $\tau = \beta = 0.2$, $\alpha = \frac{1}{3}$, $s = 0.2$, and $\delta = 0.06$.

[b] Cumulated gain/loss for region $i = (\sum_t y^i_{TSS} / \sum_t y^i_{-TSS} - 1) \times 100$. $i = 1, \ldots$.

[c] Cumulated gain/loss for the federal state

$$= \left[\sum_t (y^1 + y^2)_{-TSS} / \sum_t (y^1 + y^2)_{-TSS} - 1\right] \times 100, \quad t = 1, \ldots .$$

[d] Catch-up factor = percentage of initial gap closed by period t

$$= \{[1 \quad (y_t^2 - y_t^1)] / (y_0^2 - y_0^1)\} \times 100.$$

Endogenous growth

A case which deserves special attention is the deviation from the neoclassical assumption of constant returns in production. We could assume a broader definition of the capital stock and the productive government spendings and increase their shares of national income (α and β, respectively) to allow for $\alpha + \beta \geqslant 1$ in production function (1)

For $\alpha + \beta = 1$ the analysis for isolated regions (section 2) follows Barro's (1990) "simple model of endogenous growth" without transitional dynamics and constant growth rates. For example, the growth rate of capital intensity (and output) equals 0.5% for $\alpha = 0.6$, $\beta = 0.4 = \tau$ and the same base values as before otherwise. Therefore, income levels are diverging according to the initial gap in capital stocks (compare $y^1_{\neg TSS}$ and $y^2_{\neg TSS}$ in figure 5; note that time paths are not linear but exponential). In this case, implementation of a TSS is capable of changing the situation dramatically. Instead of diverging income levels, a catching up of the poor region to the rich one could take place (or at least the divergence is much slower than without TSS). By introduction of the TSS, not only is the level of income of the rich region decreased but also its growth rate, whereas the opposite happens in the poor region.

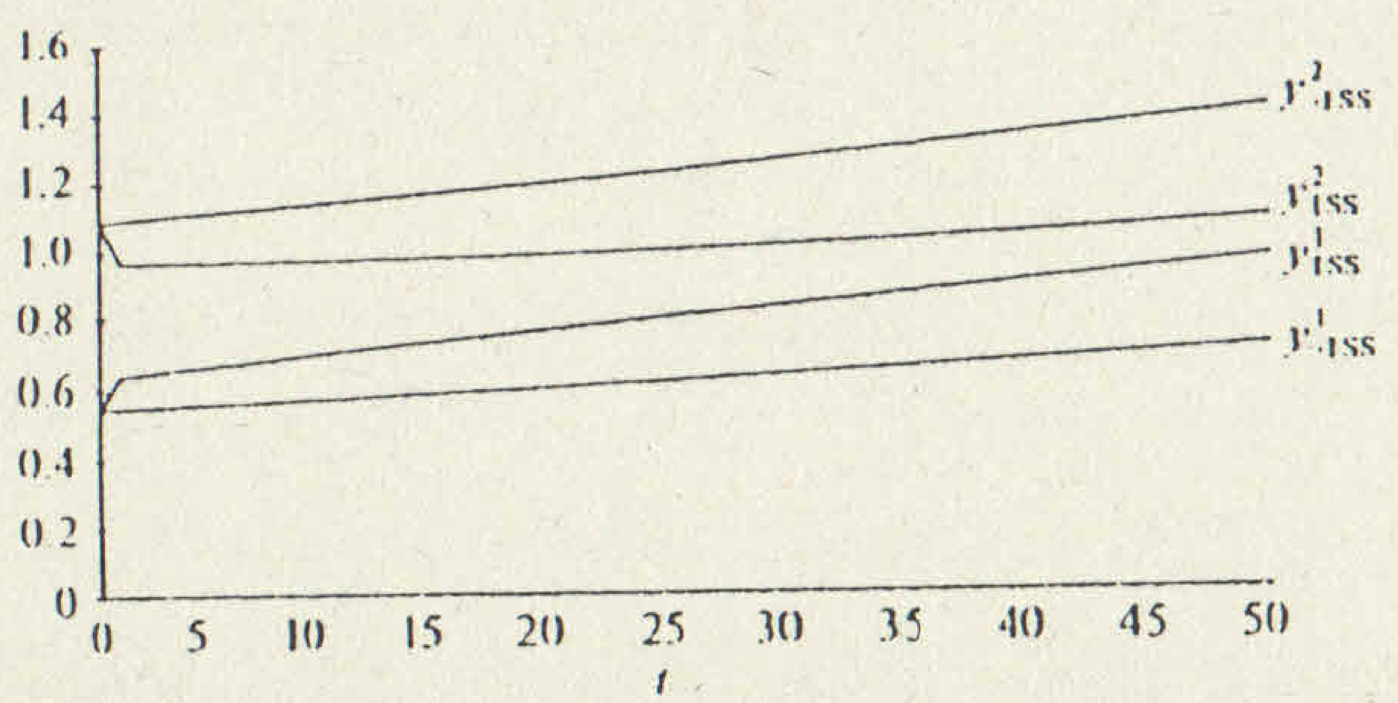

Figure 5. Development of y over time if $\alpha + \beta = 1$. $\alpha = 0.6$ and $\beta = 0.4$; moreover, $\tau = 0.4$, $s = 0.2$, and $\delta = 0.06$; $k^1_0 = 1$ and $k^2_0 = 2$, that is, $y^1_0 = 0.54$ and $y^2_0 = 1.09$.

The development of income levels according to our base case but with $\alpha = 0.6$ and $\beta = 0.4$ ($= \tau$) is shown in figure 5. The catching up covers 50% of the initial gap after 10 periods, whereas 79% of the gap is closed after 50 periods of TSS. Without TSS the regions would have diverged by 29% of the initial gap after 50 periods. Concerning relative catching up, that is, the income gap in terms of the income level of region 1, the poor region catches up by 50% in the first 2 periods and continues to close the gap, for example, to 12% after 50 periods. Without TSS the relative gap is the same (100%) forever.

After the shock of introducing the TSS where region 1's growth rate more than doubles (from 0.5% to 1.1%), whereas region 2's growth rate shrinks considerably (to 0.05%), growth rates in both regions slowly approach their steady-state value (0.5%) again. With constant retu in capital and government expenditures taken together the TSS is not profitable anymore. However, the costs in terms of aggregate foregone output for the total economy are below 3% even in the very long run and the gains for region 1 are very large (for example, 30% of cumulated output in 50 periods) as are the losses for region 2 (19%).

If we allow for $\alpha + \beta > 1$, that is, for exponentially increasing growth rates, results do not change qualitatively with respect to the case just described. However, the variables reach implausible values rather soon—a well-known problem in the 'new growth theory'.

4 Conclusions

We have shown that, even in the simple setting of an augmented Solow model, the static analysis of a TSS may be enriched considerably. There are positive and negative allocative and distributional effects and the assessment of the overall effect depends on the degree of aggregation and the time horizon considered.

The effects of a TSS on an economy in transition to a long-run equilibrium are mainly distributive. In the short run, a relatively large redistribution can be achieved without considerable decreases in income level for the aggregate economy. Moreover, depending on the parameter values used and the time horizon considered, an interregional redistribution of the tax revenue may even lead to a more efficient allocation of capital inputs. Put it differently, over time the TSS produces a positive growth effect on the region that is lagging behind, which may compensate the negative level effect on the leading region. Moreover, we can show that by levying different optimal tax rates in the regions this effect could be further enhanced (compare Kellermann and Schmidt, 1995).

In our model the TSS does not influence the long-run equilibrium of the economy. It has, however, a positive influence on the poor region's rate of convergence to the steady state and on the rate by which it is catching up to the rich one. In return, the rich region experiences a decrease of its convergence speed.

These conclusions hinge on several strong assumptions and simplifications. For instance, in our specification we do not allow for differences in technology, savings behaviour, etc. We exclude factor mobility among regions which share their tax revenues. Moreover, we designed the interregional transfers as conditional grants and left out any politico-economic considerations relating to the use of tax revenues and transfers. After all, the absolute importance of our conclusions may be qualified when taking into account these 'more realistic' conditions. However, the relative weight and the direction of the effects identified in our analysis will survive and to point out these effects was the purpose of this paper.

References

Arrow K J, Kurz M, 1970 *Public Investment, the Rate of Return, and Optimal Fiscal Policy* (Johns Hopkins University Press, Baltimore, MD)

Aschauer D A, 1989, "Is public expenditure productive?" *Journal of Monetary Economics* 23 177 – 200

Aschauer D A, 1995, "Infrastructure and macroeconomic performance: direct and indirect effects", in *The OECD Jobs Study: Investment, Productivity and Employment* (OECD, Paris) pp 85 – 101

Barro R J, 1990, "Government spending in a simple model of endogenous growth" *Journal of Political Economy* 98 S103 – S125

Barro R J, Sala-i-Martin X, 1995 *Economic Growth* (McGraw-Hill, New York)

Barro R J, Mankiw N G, Sala-i-Martin X, 1995, "Capital mobility in neoclassical models of growth" *American Economic Review* 85 103 – 115

Bayoumi T, Masson P R, 1995, "Fiscal flows in the United States and Canada: lessons for monetary union in Europe" *European Economic Review* 39 253 – 274

Bergstrom T C, Goodman R P, 1973, "Private demand for public goods" *American Economic Review* 63 280 – 296

Borcherding T E, Deacon R T, 1972, "The demand for the service of non-federal governments" *American Economic Review* 62 891 – 901

Chiang A C, 1992 *Dynamic Optimization* (McGraw-Hill, New York)

Gramlich E M, 1987, "Federalism and federal deficit reduction" *National Tax Journal* 40 299 – 313

Homburg S, 1993, "Eine Theorie des Finanzausgleichs: Finanzausgleich und Produktionseffizienz" *Finanzarchiv* 50 458 – 486

Inman R P, 1988, "Federal assistance and local services in the United States: the evolution of a new federalist fiscal order", in *Fiscal Federalism: Quantitative Studies* Ed. H Rosen (University of Chicago Press, Chicago, IL) pp 33 – 74

Kaizuka K, 1965, "Public goods and decentralization of production" *Review of Economics and Statistics* 47 118 – 120

Kellermann K, Schmidt H, 1995, "Tax sharing and convergence", Institute for Empirical Research in Economics Discussion Paper, number 9504

Mankiw N G, Romer D, Weil D N, 1992, "A contribution to the empirics of economic growth" *Quarterly Journal of Economics* 107 407 – 437

Oates W E, 1972 *Fiscal Federalism* (Harcourt, Brace, Jovanovich, New York)

Oates W E, 1979, "Lump sum intergovernmental grants have price effects", in *Fiscal Federalism and Grants-in-Aid* Eds P Mieszowski, W Oakland (Urban Institute, Washington, DC) pp 23 – 30

Ramsey F, 1928, "A mathematical theory of saving" *Economic Journal* 38 543 – 559

Richter W F, Wellisch D, 1993, "Allokative theorie eines interregionalen finanzausgleichs bei unvollständiger landrentenabsorption" *Finanzarchiv* 50 433 – 457

Seitz H, 1995, "Konvergenz: theoretische Aspekte und empirische Befunde für Westdeutsche Regionen" *Konjunkturpolitik* 41 168 – 198

Solow R M, 1956, "A contribution to the theory of economic growth" *Quarterly Journal of Economics* 70 65 – 94

Stiglitz J E, 1977, "The theory of local public goods", in *The Economics of Public Services* Eds M Feldstein, R P Inman (Macmillan, London) pp 274 – 333

Wildasin D E, 1986, *Urban Public Finance* (Harwood Academic, Chur)